SECTION 1 (50 MARKS)

Answer all questions in this section.

1. Solve for x in the equation $2\mathrm{Sin}^2x - 1 = \mathrm{Cos}^2x + \mathrm{Sin}x$ for $0^0 \le x \le 360^0$ (3marks)

S | A
T | C

$2\sin^2 x - 1 = 1 - \sin^2 x + \sin x$

$2\sin^2 x + \sin^2 x - \sin x - 1 - 1 = 0$

$3\sin^2 x - \sin x - 2 = 0$

$3\sin^2 x - 3\sin x + 2\sin x - 2 = 0$

$3\sin x(\sin x - 1) + 2(\sin x - 1) = 0$

$(\sin x - 1)(3\sin x + 2) = 0$

$\sin x - 1 = 0$ or $\frac{3\sin x}{3} = -\frac{2}{3}$

$\sin x = 1$ $\quad \sin x = -2/3$

$x = 90°$ $\quad x = 180 + 41.81 = \underline{138.19}$

$x =$ $\quad x = 360 - 41.81 = \underline{318.19}$

3

2. (a) Expand $\left(1 + \frac{3}{x}\right)^5$ up to the fifth term (2marks)

1 5 10 10 5

$1 + 5\left(\frac{3}{x}\right) + 10\left(\frac{3}{x}\right)^2 + 10\left(\frac{3}{x}\right)^3 + 5\left(\frac{3}{x}\right)^4$

$1 + \frac{15}{x} + \frac{90}{x^2} + \frac{270}{x^3} + \frac{405}{x^4}$

2

(b) Hence use your expansion to evaluate the value of $(2.5)^5$ to 3 d.p. (2 marks)

$2.5 = 1 + \frac{3}{x}$

$2.5 - 1 = \frac{3}{x}$

$1.5 = \frac{3}{x}$

$\frac{1.5x}{1.5} = \frac{3}{1.5}$

$\underline{x = 2}$

$1 + \frac{15}{2} + \frac{90}{4} + \frac{270}{8} + \frac{405}{16}$

$= 90.0625$

$= \underline{90.063}$

2

3. Complete the table below for $y = 8 - 2x - x^2$ for $-4 \le x \le 2$.

x	-4	-3	-2	-1	0	1	2
y	0	5	8	9	8	5	0

Hence use trapezium rule with six strips to find the area of the region bounded by the curve and the x- axis. (3marks)

$A = \frac{1}{2} \times 1 \left\{ 0 + 0 + 2(5 + 8 + 9 + 8 + 5) \right\}$

$= \frac{1}{2} \times 2(35)$

$= \underline{35}$

3

4. Make p the subject of the formula (3 marks)

$$e = \sqrt{\frac{p-3u}{y-3xp}}$$

$\frac{e^2}{1} = \frac{p-3u}{y-3xp}$

$e^2(y-3xp) = p-3u$

$e^2y - 3e^2xp = p-3u$

$e^2y + 3u = p + 3e^2xp$

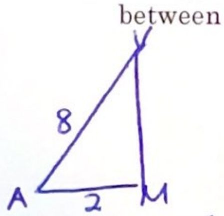
$p + 3e^2xp = e^2y + 3u$

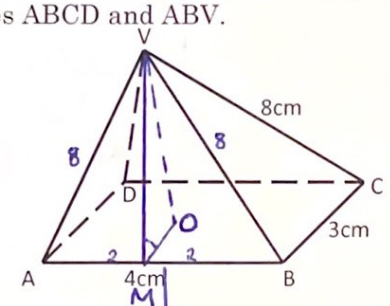
$p(1+3e^2x) = e^2y+3u$

$p = \frac{e^2y+3u}{1+3e^2x}$

2

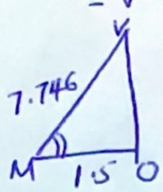
5. The figure below shows a rectangular based right pyramid. Find the angle between the planes ABCD and ABV. (3marks)





$VM = \sqrt{8^2-2^2}$

$= \sqrt{60} = 7.746$



$\cos M = \frac{1.5}{7.746}$

$\cos M = 0.19365$

$\cos^{-1} 0.19365$

$= 78.83^\circ$

3

6. ~~A~~ An object A of area $10cm^2$ is mapped onto its image B of area $60cm^2$ by a transformation whose matrix is given by P= $\begin{Bmatrix} x & 4 \\ 3 & x+3 \end{Bmatrix}$. Find the possible values of x (3 marks)

Area Scale factor $= \frac{60}{10} = 6$

$\begin{bmatrix} x & 4 \\ 3 & x+3 \end{bmatrix}$

Determinant

$x(x+3) - 12 = x^2+3x-12$

$x^2+3x-12 = 6$

$x^2+3x-12-6=0$

$x^2+3x-18=0$

$x^2+6x-3x-18=0$

$x(x+6)-3(x+6)=0$

$(x+6)(x-3)=0$

$x+6=0$ $x-3=0$

$x=-6$ $x=3$

3

7. Find the value of x in the equation
$\log_{10} 5 - 2 + \log_{10}(2x+10) = \log_{10}(x-4)$ (3marks)

$\log_{10} 5 - 2\log_{10} 10 + \log_{10}(2x+10) = \log_{10}(x-4)$

$\log_{10} 5(2x+10) - \log_{10} 100 = \log_{10}(x-4)$

$\log_{10}\left(\frac{10x+50}{100}\right) = \log_{10}(x-4)$

$\frac{10x+50}{100} = \frac{x-4}{1}$

$10x+50 = 100(x-4)$

$10x+50 = 100x-400$

$50+400 = 100x-10x$

$\frac{450}{90} = \frac{90x}{90}$

$\underline{\underline{x=5}}$

3

8. The data below shows marks obtained by 10 students in a test.
71, 55, 69, 45, 65, 57, 71, 82, 55, 50. Calculate the standard deviation using an assumed mean of 60. (3marks)

x	x − 60	$(x-60)^2$
71	11	121
55	−5	25
69	9	81
45	−15	225
65	5	25
57	−3	9
71	11	121
82	22	484
55	−5	25
50	−10	100
	20	1216

$\sqrt{\frac{\Sigma(x-a)^2}{\Sigma f} - \left[\frac{\Sigma(x-a)}{\Sigma f}\right]^2}$

$\sqrt{\frac{1216}{10} - \left(\frac{20}{10}\right)^2}$

$= \sqrt{121.6 - 4}$

$= \sqrt{117.6}$

$= \underline{\underline{10.84}}$

3

9. Evaluate by rationalizing the denominator and leaving your answer in surd form. (3marks)

$$\frac{\sqrt{8}}{1+Cos45^0}$$

$\frac{(\sqrt{8})\sqrt{2}}{\left(1+\frac{1}{\sqrt{2}}\right)\sqrt{2}}$

$\frac{\sqrt{16}}{\sqrt{2}+1} = \frac{4}{\sqrt{2}+1}$

$\frac{4}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$\frac{4\sqrt{2}-4}{2-1}$

$\underline{\underline{4\sqrt{2}-4}}$

3

10. The position vectors fof points A and B are $5i+4j-6k$ and $2i-2j$ respectively. A point X divides AB in the ratio -3: 5. Find the coordinates of X. (3marks)

$\vec{OA} = \begin{pmatrix} 5 \\ 4 \\ -6 \end{pmatrix}$ $\vec{OB} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$

$AX : XB$

$-3 : 5$

$\vec{OX} = \frac{-3}{2}\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} + \frac{5}{2}\begin{pmatrix} 5 \\ 4 \\ -6 \end{pmatrix}$

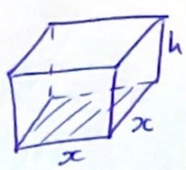
$= \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 12.5 \\ 10 \\ -15 \end{pmatrix}$

$\vec{OX} = \begin{pmatrix} 9.5 \\ 13 \\ -15 \end{pmatrix}$

$X(9.5, 13, -15)$

3

11. A closed box has a square base of side x metres and its height h metres. The total surface area of the box is 24m².
(a) Find the expression of h in terms of x.



$S.A = 2x^2 + 4xh$

$2x^2 + 4xh = 24$

$4xh = 24 - 2x^2$

$h = \frac{24 - 2x^2}{4x}$

$h = \frac{24}{4x} - \frac{2x^2}{4x}$

$h = \frac{6}{x} - \frac{x}{2}$

2

(b) Hence find the value of x that would make the volume of the box maximum. (4 marks)

$V = b.a \times h.$

$= x^2 \times h$

$= hx^2$

$V = hx^2$

$= x^2\left(\frac{6}{x} - \frac{x}{2}\right)$

$V = 6x - \frac{x^3}{2}$

$\frac{dV}{dx} = 6 - \frac{3}{2}x^2$

$6 - \frac{3x^2}{2} = 0$

$2 \times 6 = \frac{3x^2}{\cancel{2}} \times \cancel{2}$

$\frac{12}{3} = \frac{3x^2}{3}$

$\sqrt{4} = \sqrt{x^2}$

$2 = x$

2

12. M varies directly as D and as the cube of V. Calculate the percentage change in M when V is increased by 10% and D is reduced by 10%. (3marks)

$M = KDV^3$

$M_1 = K(0.9D)(1.1V)^3$

$M_1 = 0.9 \times 1.1^3 KDV^3$

$M_1 = 1.1979 KDV^3$

$= 1.1979M$

$\%\text{ Change} = \frac{1.1979M - M}{M} \times 100$

$= 0.1979 \times 100$

$= 19.79\%$

3

13. Find the value of t if the gradient of the graphs of the functions $y = x^2 - x^3$ and $y = x - tx^2$ are equal at $x = \frac{1}{3}$. (3marks)

$y = x^2 - x^3$

$dy/dx = 2x - 3x^2$

$y = x - tx^2$

$dy/dx = 1 - 2tx$

$x = 1/3$

$dy/dx = 2(1/3) - 3(1/3)^2$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

$dy/dx = 1 - 2t(1/3)$

$= 1 - \frac{2}{3}t$

$1/3 = 1 - \frac{2}{3}t$

$\frac{2}{3}t = 1 - 1/3$

$\frac{2}{3}t = 2/3$

$t = 1$

3

14. The image of a point A, under the transformation represented by the matrix $T = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ is A' (-2, 4)Find the coordinates of A (3marks)

$A(a, b) \quad A'(-2, 4)$

$T = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$

Det $2 - 0 = 2$

$T^{-1} = \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0.5 \\ 0 & 0.5 \end{pmatrix}$

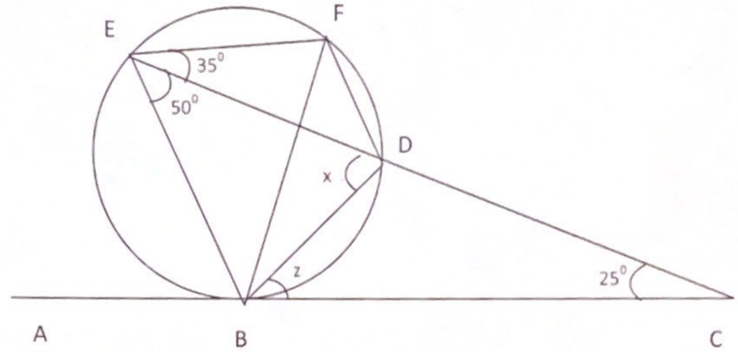
$\begin{pmatrix} 1 & 0.5 \\ 0 & 0.5 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$A(0, 2)$

3

15. In the figure below, ABC is a tangent at B and CDE is a straight line.

$\angle BED = 50^0$, $\angle DEF = 35^0$ and $\angle ECB = 25^0$



Calculate the values of x and z. (2marks)

$z = 50^\circ$

$x = 50 + 25$

$= 75^\circ$

2

16. The equation of a circle is given by $4x^2 + 4y^2 - 8x + 2y - 7 = 0$

Determine the coordinates of the centre of the circle. (3marks)

$\frac{4x^2}{4} - \frac{8x}{4} + \frac{4y^2}{4} + \frac{2y}{4} = \frac{7}{4}$

$x^2 - 2x + \left(\frac{-2}{2}\right)^2 + y^2 + \left(\frac{1}{4}\right)^2 = \frac{7}{4} + \left(\frac{-2}{2}\right)^2 + \left(\frac{1}{4}\right)^2$

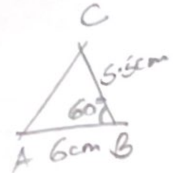
$(x-1)^2 + (y + 1/4)^2 = \frac{7}{4} + 1 + \frac{1}{16}$

$(x-1)^2 + (y + 1/4)^2 = \frac{28+16+1}{16}$

$(x-1)^2 + (y + 1/4)^2 = \frac{45}{16}$

Centre $(1, -1/4)$

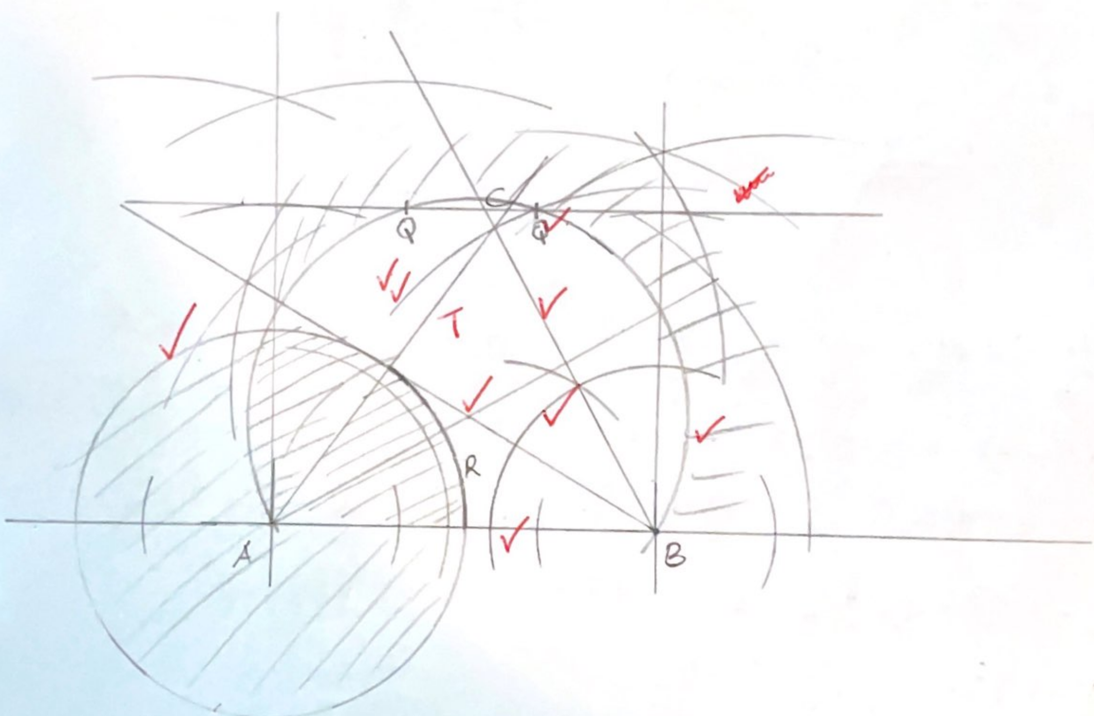
3



SECTION II (50 MARKS)

*Answer **only five** questions from this section*

17. (a) Using a ruler and a pair of compasses only construct triangle ABC in which AB = 6cm, BC = 5.5cm and angle ABC = 60^0. Measure AC. (3marks)



(b) On the same side of AB as C, determine the locus of a point P such that angle APB = 60^0 (2marks)

In the triangle

(c) Construct the locus of R such that AR = 3cm (1mark)

(d) Identify the region T such that AR≥ 3cm and angleAPB≥ $60^0$by shading the unwanted part. (2marks)

(e) Determine point Q such that area of AQB is half the area of ABC and that Angle AQB = 60^0. (2marks)

Area of Δ ABC = $\frac{1}{2} \times 6 \times 5.5 \times \sin 60$

$= 14.289$

$\frac{14.2894}{3} = \frac{1}{\cancel{2}} \times \frac{\cancel{6}^{3} \times h}{3}$

$h = 4.76 \simeq 5cm.$

18. A sequence is formed by adding corresponding terms of an AP and GP. The first, second and third terms of the sequence formed are 14, 34 and 78 respectively.

(a) Given that the common ratio of the GP is 3;

(i) Find the first term of the AP and GP and the common difference of the AP. (2marks)

AP: $a \quad a+d \quad a+2d$

GP $x \quad xr \quad xr^2$ +

$x \quad 3x \quad 9x$

$a+x=14$

$2(a+d+3x=34)$

$a+2d+9x=78$

$2a+2d+6x=68$

$a+2d+9x=78$ −

$a-3x=-10$

$a+x=14$

$\frac{-4x}{-4}=\frac{-24}{-4}$

$\underline{x=6}$

$a+x=14$

$a+6=14$

$\underline{a=8}$

2

(ii) Find the sixth term and the sum of the first six terms of the sequence. (3marks)

14, 34, 78

$T_6 = a+5d+3^5x$

$\cancel{=8+}$

$a+d+3x=34$

$8+d+18=34$

$d=34-26$

$\underline{d=8}$

$T_6 = a+5d+243x$

$= 8+5(8)+243\times6$

$= \underline{1506}$

$T_5 = a+4d+81x$

$= 8+4(8)+81\times6$

$= \underline{526}$

$T_4 = a+3d+27x$

$8+3(8)+27\times6 = \underline{194}$

S_6

$14+34+78+194$

$+526+1506$

$= \underline{2352}$

3

(b) The second and third terms of a geometric progression are 24 and 12(x + 1) respectively.

Find the whole number value of x and hence the first term given the sum of the first three terms of the progression is 76. (5marks)

$ar = 24$

$ar^2 = 12(x+1)$

$r = \frac{12(x+1)}{24} = \frac{x+1}{2}$

$ar = 24$

$a\left(\frac{x+1}{2}\right) = 24$

$a = \frac{24\times2}{x+1} = \frac{48}{x+1}$

$\left(\frac{48}{x+1} + 24 + 12(x+1) = 76\right)(x+1)$

$48+24(x+1)+12(x+1)^2 = 76(x+1)$

$48+24x+24+12(x^2+2x+1) = 76x+76$

$72+24x+12x^2+24x+12-76x-76=0$

$\frac{12x^2}{4} - \frac{28x}{4} + \frac{8}{4} = 0$

$3x^2-7x+2=0$

$3x^2-6x-x+2=0$

$3x(x-2)-1(x-2)=0$

$(x-2)(3x-1)=0$

$x=2$ or $x=\frac{1}{3}$

$\underline{x=2}$

5

10

19. Income tax rate are as shown below.

Income (k£ p.a)	Rate (Ksh per £)
1- 4200	2
4201 - 8000	3
8001 - 12600	5
12601 – 16800	6
16801 and above	7

Omari pays Sh. 4000 as P.A.Y.E per month. He has a monthly house allowance of Ksh. 10800 and is entitled to a personal relief of Ksh. 1,100 per month. Determine;

(i) his gross tax p.a in Ksh (2marks)

4000
+ 1100
5100 × 12 = 61,200

2

(ii) his taxable income in k£ p.a (4marks)

4200 × 2 = 8400
3800 × 3 = 11,400
4600 × 5 = 23,000
42,800
x × 6 = 18,400
$x = \frac{18400}{6}$ = 3066.67

12,600
3066.67
£ 15,666.67

4

(iii) his basic salary in Ksh. p.m (2marks)

$\frac{15,666.67 \times 20}{12}$ = 26,111.11
− 10,800
15,311.11

2

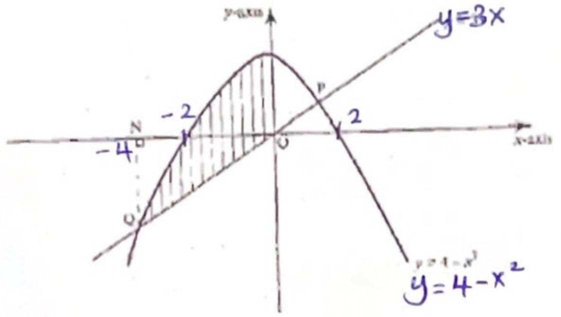
(iv) his net salary per month (2marks)

26,111.11
− 4000
22,111.11

2

10/10

20. The diagram below shows a sketch of the line y = 3x and the curve $y = 4 - x^2$ intersecting at point P and Q.



(a) Find the co-ordinates of P and Q (4marks)

$4 - x^2 = 3x$
$0 = x^2 + 3x - 4$
$x^2 + 4x - x - 4 = 0$
$x(x+4) - 1(x+4) = 0$
$(x+4)(x-1) = 0$

$x = -4$ $x = 1$
$y = 3(-4)$ $y = 3(1)$
$= -12$ $= 3$
$Q(-4, -12)$ $P(1, 3)$

4

(b) Given that QN is perpendicular to the x-axis at N, calculate
(i) the area bounded by the curve $y = 4 - x^2$, the x-axis and line QN. (2marks)

$4 - x^2 = 0$
$(2-x)(2+x) = 0$
$x = 2$, $x = -2$

$\int_{-4}^{-2} 4 - x^2\,dx = \left[4x - \frac{x^3}{3}\right]_{-4}^{-2}$

$\left(4(-2) - \frac{(-2)^3}{3}\right) - \left(4(-4) - \frac{(-4)^3}{3}\right)$
$= -5\frac{1}{3} - 5\frac{1}{3}$
$= -10\frac{2}{3}$ $= 10\frac{2}{3}$

2

(ii) the area of the shaded region that lies below the x - axis (2marks)

$\int_{-4}^{0} 3x\,dx = \frac{3x^2}{2}\Big|_{-4}^{0} = \left[\frac{3(0)^2}{2}\right] - \frac{3(-4)^2}{2} = 0 - 24$

$24 - 10\frac{2}{3}$
$= 13\frac{1}{3}$

2

(iii) the area of the region enclosed by the curve $y = 4 - x^2$, the line y = 3x and the y-axis (2marks)

$\int_{-2}^{0} 4 - x^2\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{0}$

$\left[4(0) - \frac{(0)^3}{3}\right] - \left[4(-2) - \frac{(-2)^3}{3}\right]$

$0 - -5\frac{1}{3} = 5\frac{1}{3}$

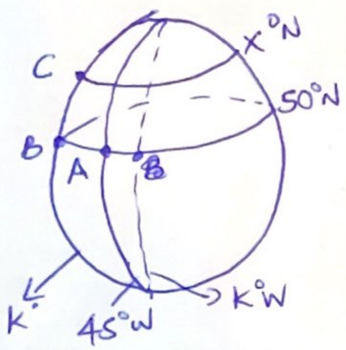
$5\frac{1}{3} + 13\frac{1}{3}$
$= 18\frac{2}{3}$

2

10

21. The positions of two towns A and B are (50^0N, 45^0W) and (50^0N, K^0W) respectively. It takes a plane 5 hours to travel from A to B at an average speed of 800knots. The same plane takes $1\frac{1}{2}$ hours to travel from B to another town C at the same average speed. Given that C is to the north of B, calculate to the nearest degree,

(a) The value of K (4marks)



$D = S \times T$
$= 800 \times 5$
$= 4000$nm

$1^\circ = 60 \cos 50^\circ$
$? \quad 4000$

$\frac{4000 \times 1}{60 \cos 50}$

$= 103.7^\circ$

$K - 45 = 103.7$
$K = 103.7 + 45$
$= 148.7^\circ$ W

4

(b) The latitude of C (3marks)

$D = S \times T$
$= 800 \times \frac{3}{2} = 1200$nm

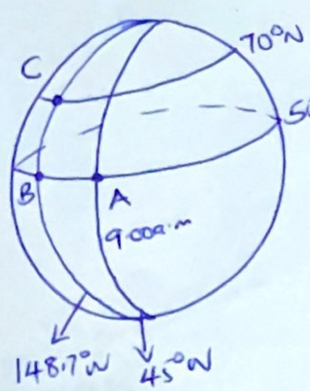
$1^\circ = 60$nm
$? \quad 1200$nm

$\frac{1200 \times 1}{60} = 20^\circ$

$X - 50 = 20$
$X = 70^\circ$N

3

(c) If the plane started from A at 9.00am and flew to C through B, find the local time at C when the plane arrived there. (3marks)



Distance A - B - C
$= 4000$nm $+ 1200$
$= 5200$nm

$T = \frac{D}{S}$
$= \frac{5200}{800}$
$= 6\frac{1}{2}$ hrs

$\theta = 103.7^\circ$
$1^\circ = 4$min
$103.7 = 103.7 \times 4 = \frac{414.8}{60}$
$= 6$hrs 55min

3

9 : 00
$-$ 6 : 55
02 : 05a.m.
$+$ 6 : 30
8 : 35a.m at C

10

22. (a) Complete the table below for the equation $y = x^3 + 2x^2$ to 2 d.p(2marks)

X	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5
$2x^2$	18	12.5	8	4.5	2	0.5	0	0.5	1	4.5
X^3	-27	-15.63	-8	-3.38	-1	-0.13	0	0.13	1	3.38
y	-9	-3.13	0	1.12	1	0.37	0	0.63	2	7.88

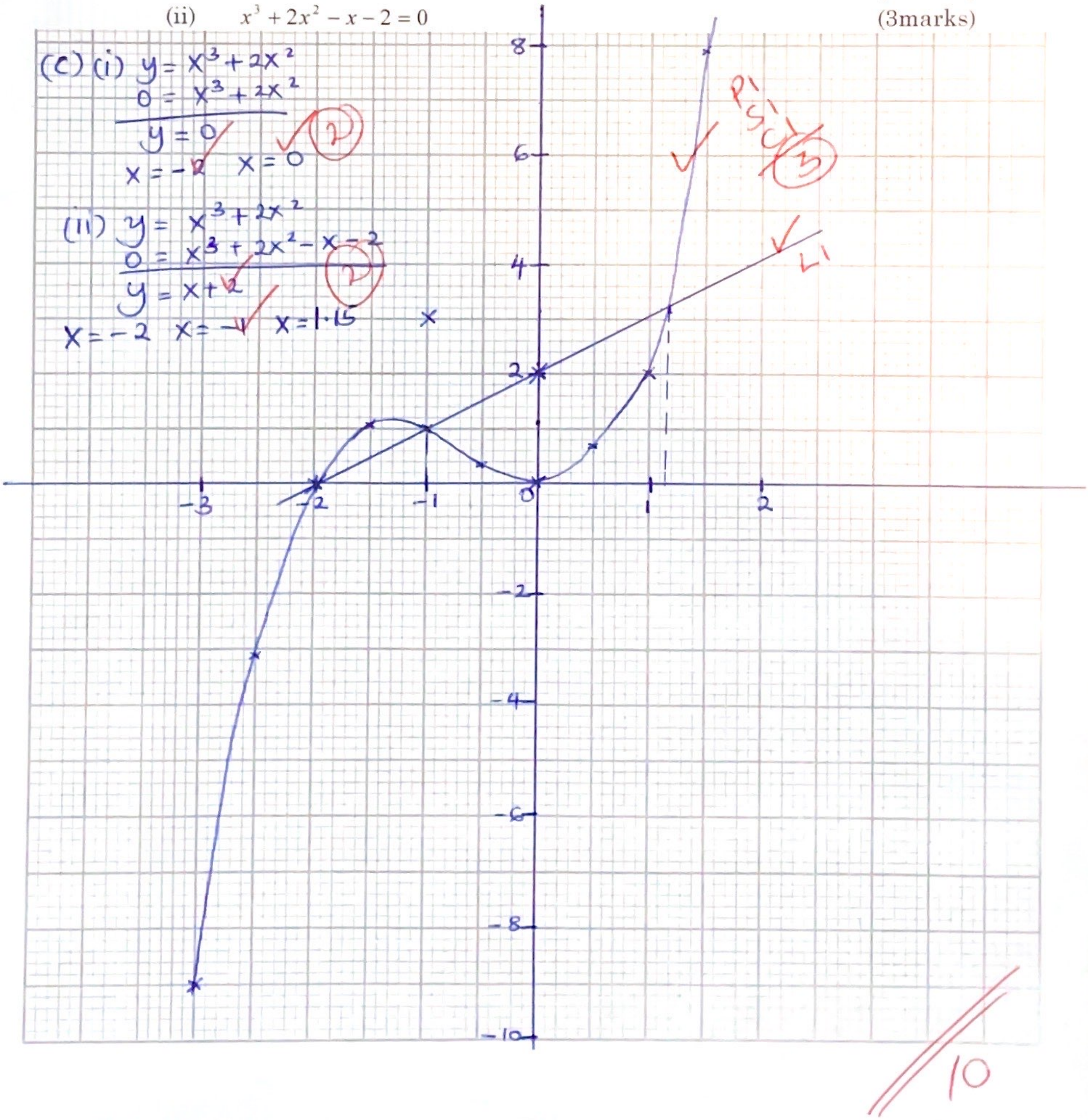
2

(b) On the grid provided, draw the graph of $y = x^3 + 2x^2$ for $-3 \leq x \leq 1.5$. Take a scale of 2cm to represent 1 unit on the x- axis and 1cm to represent 1unit on the y – axis. (3marks)

(c) Use your graph to solve

(i) $x^3 + 2x^2 = 0$ (2marks)

(ii) $x^3 + 2x^2 - x - 2 = 0$ (3marks)



23. The table below shows the marks obtained by 47 students in a mathematics test.

Marks	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55	56 - 60
No. of candidates	4	6	12	15	8	2
cf	4	10	22	37	45	47

(a) On the grid provided, draw a cumulative frequency curve. (3marks)

(b) Use your graph to estimate

(i) The median $47/2 = 23.5$ 46 (2marks)

(ii) The semi interquartile range (2marks)

(c) In order to pass the test a candidate had to score more than 40 marks. Calculate the percentage of candidates who passed. (3marks)

(i) Median $\frac{47}{2} = 23.5 \Rightarrow 46$ 2

(ii) $Q_3 = \frac{3}{4} \times 47 = 35.25 \Rightarrow 50$ 2

$Q_1 = \frac{1}{4} \times 47 = 11.75 \Rightarrow 41$

$\frac{Q_3 - Q_1}{2} = \frac{50 - 41}{2} = 9/2 = \underline{4.5}$

(c) $47 - 9 = 38$

$\frac{38}{47} \times 100 = \underline{80.85\%}$ 3

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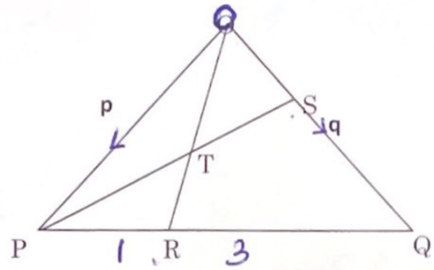
24. In the triangle OPQ below, **OP** = **p** and **OQ** = **q**. R is a point on PQ such that PR: RQ = 1 : 3 and 5OS = 2 OQ. PS intersects OR at T.



(a) Express in term of **p** and **q**

(i) **OS** $= \frac{2}{5}\vec{OQ} = \frac{2}{5}q$ (1) (1mark)

(ii) **PQ** $= q - p.$ (1) (1mark)

(iii) **OR** $= \frac{1}{4}q + \frac{3}{4}p$ (2) (2mark)

(b)Given that **OT**= h**OR** and **PT** = k**PS**. Determine the values of h and k. (6marks)

(i) $\vec{OT} = h\,\vec{OR}$
$= h\left(\frac{1}{4}q + \frac{3}{4}p\right)$
$= \frac{h}{4}q + \frac{3h}{4}p$

(ii) $\vec{OT} = \vec{OP} + \vec{PT}$
$= p + k\left(-p + \frac{2}{5}q\right)$
$= p - kp + \frac{2k}{5}q$
$= p(1-k) + \frac{2k}{5}q.$

(6)

$\frac{h}{4}q + \frac{3h}{4}p = p(1-k) + \frac{2k}{5}q$

$\frac{3h}{4} = 1-k$

$4 \times \frac{h}{4} = \frac{2k}{5} \times 4$

$h = \frac{8}{5}k$

$\frac{3}{\cancel{4}}\left(\frac{\cancel{8}^2}{5}k\right) = 1-k$

$\frac{6}{5}k + \frac{5}{5}k = 1$

$\frac{11}{5}k = 1$

$k = \frac{5}{11}$

$h = \frac{8}{\cancel{5}}\left(\frac{\cancel{5}}{11}\right)$

$h = \frac{8}{11}$

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